

Open book, open notes, calculator allowed. Write your name on this exam, you must turn it in, but you will get it back! You may write on this exam. Write all answers on your scantron form.

(1) Suppose that 7 different elements are selected at random from the set  $\{1,2,3,4,5,6,7,8,9,10,11,12\}$ . Which arrangement of numbers within boxes could be part of an explanation as to why some pair of the selected numbers will have a difference of 3?

- (A) 

1
---

3
---

5
---

7
---

9
---

11
----
- (B) 

1,4, 7,10
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2,5, 8,11
--------------

3,6, 9,12
--------------
- (C) 

1,4
-----

2,5
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3,6
-----

4,7
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5,8
-----

6,9
-----
- (D) 

1,4
-----

2,5
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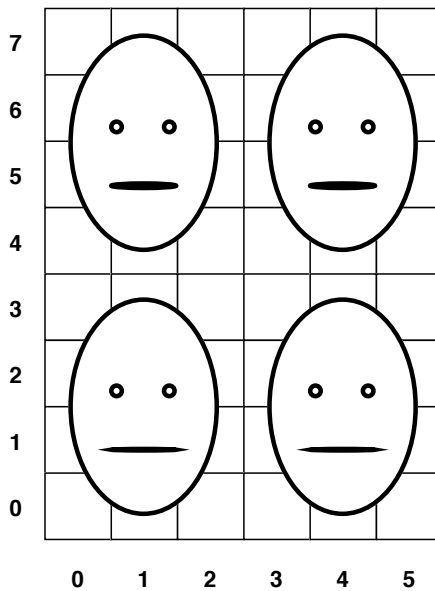
3,6
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7,10
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8,11
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9,12
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(2) If this pattern continues throughout the plane, which shows the correct placement of the face in the boxes below?

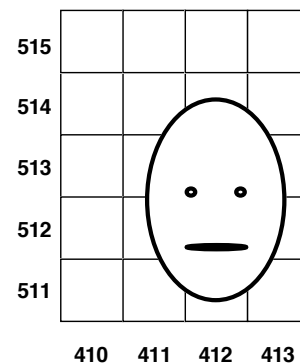
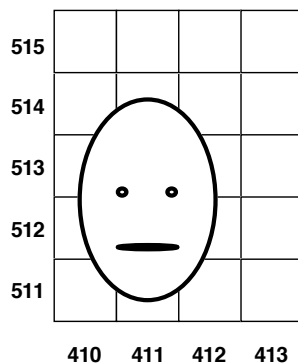
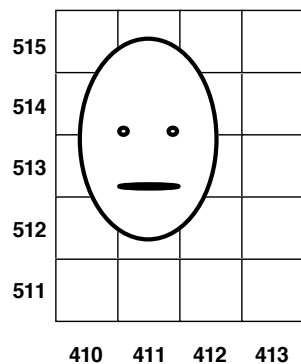
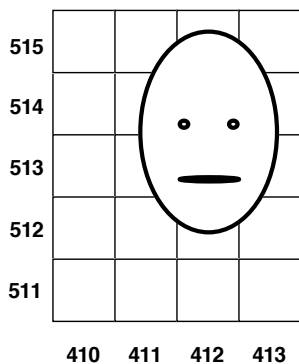


(A)

(B)

(C)

(D)



(3)

(4) [Take-away game] Suppose you play the following game with an opponent: you start with a pile of 100 bottle tops. Each of you may remove 1,2,3,4,5,6, or 7 bottle tops on each turn. The player who removes the last bottle top is the winner. Which player has a winning strategy?

- (A) The first player    (B) The second player    (C) Neither    (D) Both

(5) In the problem in (4), if the game begins with 190 bottle tops, then the first player has a winning strategy in which the first move is to take this number of bottle tops:

- (A) 8    (B) 7    (C) 6    (D) 5

(6)  $12 \equiv 2 \pmod{\quad}$ . Which of the following make this statement true if the blank is filled with any number in the list?

- (A) 3, 4, or 6                      (B) 4 or 12                      (C) 2, 5, or 10                      (D) 2, 4, or 6

(7) Which is correct?

- (A) The prime numbers begin 1,2,3,5,7,9,11,..., and a different sequence we studied begins 1,1,2,3,5,8,...  
(B) The prime numbers begin 1,2,3,5,7,9,11,..., and a different sequence we studied begins 2,3,5,7,11,...  
(C) The prime numbers begin 2,3,5,7,11,..., and a different sequence we studied begins 1,1,2,3,5,8,...  
(D) The prime numbers and another sequence we studied begin with the same 3 numbers.

(8) Logic.

On Monday Alex sold a printer to Bill for \$80, Bill sold a phone to Cora for \$70, and Cora sold an iPod to Alex for \$60.

On Tuesday Bill sold the printer to Cora for \$100, Cora sold the phone to Alex for \$90, and Alex sold the iPod to Bill for \$80 .

On Wednesday Cora sold the printer to Alex for \$50, Alex sold the phone to Bill for \$70, and Bill sold the iPod to Cora for \$60.

At the end of the day on Wednesday, who had lost or made how much from these sales?

- (A) Alex made 20, Bill came out even, and Cora made 20  
(B) Alex made 30, Bill came out even, and Cora lost 30  
(C) Alex came out even, Bill made 30, and Cora lost 30  
(D) Alex made 10, Bill lost 10, and Cora came out even


(9) Find the check digit for the ISBN 0-486-43233-\_\_

- (A) 0    (B) X    (C) 6    (D) 5

(10) The Lucas numbers are similar to the Fibonacci numbers: 1,3,4,7,11,18,29,47, ..., in which each number is also the sum of the preceding two, but the sequence begins with 1 and 3. About 2% of all spiraling plants have been found to exhibit this sequence. Which is the general formula for the "even case":

$$\begin{array}{ll} (1)(4) = 3^2 - 5 & (L_1)(L_3) = L_2^2 - 5 \\ (3)(7) = 4^2 + 5 & (L_2)(L_4) = L_3^2 + 5 \\ (4)(11) = 7^2 - 5 & (L_3)(L_5) = L_4^2 - 5 \\ (7)(18) = 11^2 + 5 & (L_4)(L_6) = L_5^2 + 5 \end{array}$$

General pattern or formula when k is an even number:

$$\begin{array}{ll} (A) (L_k)(L_{k+2}) = (L_{k+1})(L_{k+3}) & (B) (L_k)(L_{k+2}) = L_{k+1}^2 - 5 \\ (C) (L_k)(L_{k+2}) = L_{k+1}^2 + 5 & (D) (L_{k-2})(L_k) = L_{k+2}^2 - 5 \end{array}$$

(11) The smallest "counterexample" to the incorrect statement that every odd number greater than 3 can be written as the sum of two primes is which of the following? (That is, the smallest odd number that cannot be written as the sum of two primes).

- (A) 7 (B) 9 (C) 11 (D) 13

(12) If there were an infinite number of odd prime numbers that could be written as the sum of two primes, then the \_\_\_\_\_ Conjecture would have to be true. (Hint: most primes are odd. The only way for two whole numbers to add up to an odd number is if one of them is even!)

- (A) Goldbach (B) Twin Prime (C) Fermat (D) Prime Number

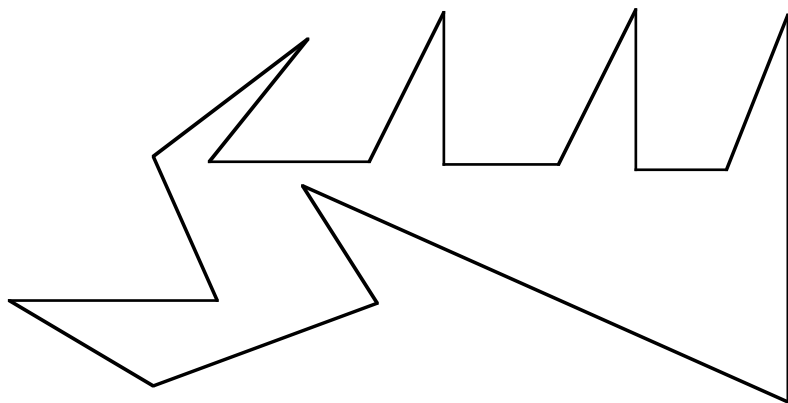
(13) An antique store uses 3 kinds of packaging to ship small objects, all rectangular boxes that are all one inch deep:

8 inches by 7 inches, 9 inches by 6 inches, and 10 inches by 5 inches. They plan to ship a long, thin knife that is 11 inches long, and less than 1 inch wide at all points. Which, if any, of the boxes will it fit in?

- (A) The 8 by 7 box and the 9 by 6 box, but not the 10 by 5 box  
 (B) The 8 by 7 box, but not the 9 by 6 box and not the 10 by 5 box  
 (C) The 9 by 6 box, but not the 8 by 7 box and not the 10 by 5 box  
 (D) The 10 by 5 box, but not the 9 by 6 box and not the 8 by 7 box

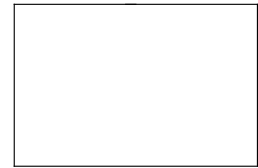
(14) The minimum number of video cameras necessary to place at corner points in order to guard this art gallery is:

- (A) 3 (B) 4 (C) 5 (D) 6



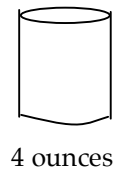
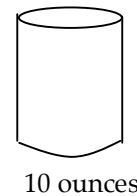
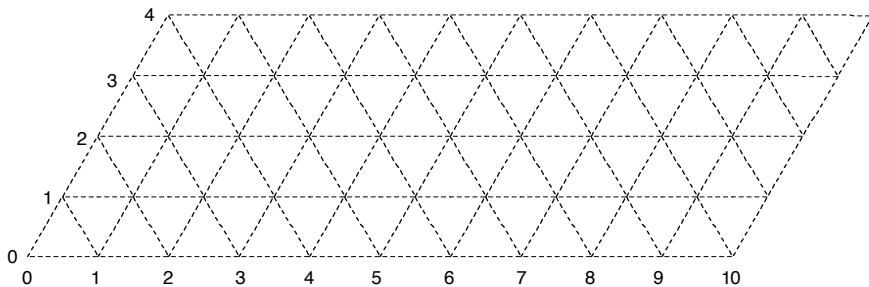
(15) (a) You have a drawer full of an uncoordinated jumble of a huge number of red, white, blue, green, and black socks. The electricity has gone off again, it is night, and you need to pick out a number of socks of the same color. What is the least number of socks you must take to guarantee that 3 of them are the same color?  
 (A) 6 (B) 8 (C) 11 (D) 16

(16) A number of Starbucks stores are to be located in this 2 mile by 3 mile rectangular area. What is the minimum number that will guarantee that at least two Starbucks are within 1.5 miles of each other?  
 (A) 4 (B) 5 (C) 6 (D) 7



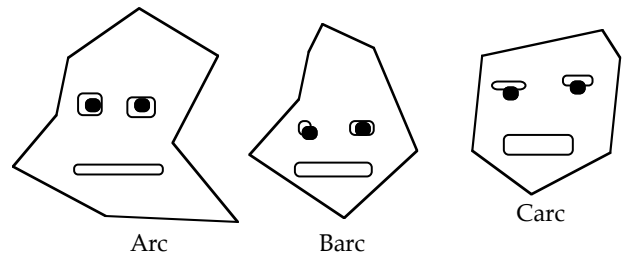
(17) Which best describes this statement: The RSA code could be broken if we knew how to easily factor the products of two very large prime numbers.  
 (A) True (B) False (C) The RSA code has nothing to do with prime numbers  
 (D) The RSA code has already been broken and it was not done by factoring

(18) You have two measuring cups. One holds exactly 4 ounces, and the other holds exactly 10 ounces. There are no markings on the cups and you are not able to mark the cups at all, nor can you "tilt" the cups so that you measure half the amount. You are given a huge bucket of water. Which of the following amounts is it possible to measure exactly, by pouring back and forth, filling or emptying the cups? (Assume a "step" is filling a cup, emptying a cup, or pouring from one cup to another.)  
 (A) Any whole number of ounces from 1 to 10, and to get 2 ounces the minimum number of steps is 4  
 (B) Any even number of ounces from 2 to 10, and to get 2 ounces the minimum number of steps is 2  
 (C) Any even number of ounces from 2 to 10, and to get 2 ounces the minimum number of steps is 4  
 (D) Any even number of ounces from 2 to 10, and to get 2 ounces the minimum number of steps is 6



(19) A well known conjecture about primes says that every even number greater than 2 is the sum of 2 primes. 42 is the sum of how many pairs of primes?  
 (A) One pair (B) Two pairs (C) Three pairs (D) Four pairs

(20) Three space aliens Arc, Barc, and Carc stand before you. One of them has stolen all the candy in the entire world. Fortunately you know that exactly one of them is lying, and so you can figure out who has taken the candy. Who is it?



Carc says, "Arc did not take the candy!"

Barc says, "I did not do it."

Arc says, "I did not do it."

(A) Arc (B) Barc (C) Carc (D) None of them did it!